

MIT LIBRARIES


DUPL



3 9080 02618 3209







Digitized by the Internet Archive  
in 2011 with funding from  
Boston Library Consortium Member Libraries

<http://www.archive.org/details/interestgroupsel00snyd>



HB31  
.M415  
no.05-03

DEWEY

Massachusetts Institute of Technology  
Department of Economics  
Working Paper Series

**INTEREST GROUPS & THE ELECTORAL CONTROL  
OF POLITICIANS**

James M. Snyder, Jr.  
Michael M. Ting

Working Paper 05-3  
February 3, 2005

Room E52-251  
50 Memorial Drive  
Cambridge, MA 02142

This paper can be downloaded without charge from the  
Social Science Research Network Paper Collection at  
<http://ssrn.com/abstract=661582>



# INTEREST GROUPS AND THE ELECTORAL CONTROL OF POLITICIANS<sup>1</sup>

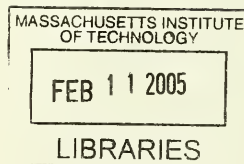
James M. Snyder, Jr.

Departments of Political Science and Economics  
Massachusetts Institute of Technology

Michael M. Ting

Department of Political Science and SIPA  
Columbia University

February 3, 2005



<sup>1</sup>We gratefully acknowledge the financial support of National Science Foundation Grant SES-0079035. We thank David Austen-Smith, Clare Leaver, and panel participants at the 2004 Stony Brook Political Economy Conference and 2004 annual meetings of the Midwest Political Science Association, Society for Social Choice and Welfare, and American Political Science Association for useful comments and discussion.

## Abstract

Are elections used to select good candidate types or to reward performance? These goals can be incompatible because the retention of good types prevents citizens from using votes to discipline incumbents. We develop a model of repeated electoral competition that examines both incentives. In each period, a randomly-drawn interest group attempts to “buy” an incumbent’s policy choice, and a voter chooses whether to replace the incumbent with an unknown challenger. The model predicts that “above average” incumbents face little discipline, but others are disciplined increasingly—and re-elected at a higher rate—as the interest group becomes more extreme. Extensions of the model consider term limits, long-lived groups, and multiple groups.

## 1. Introduction

Interest group politics is one of the most important topics in political economy and political science. However, while theorists have been analyzing formal models of interest group politics for more than thirty years, one aspect of the problem remains underdeveloped: How should strategic voters vote when they know that interest groups are trying to skew policies in ways the voters do not like? This issue has been overlooked because existing models focus on the calculations and strategic interactions of interest groups and politicians. As a result, these models treat voters as a black box, or at best as myopic actors that respond only to the short-run campaign promises of the current election.<sup>1</sup>

One obvious place to turn is the work on political agency, which focuses on the calculations voters and politicians make in a principal-agent framework. That literature goes back approximately as far as the interest group literature – Barro (1973) vs. Stigler (1971) – and has produced important insights about the possibilities and limits of using elections to control the behavior of politicians and/or select “good” types of politicians.<sup>2</sup> None of the models in this literature, however, explicitly incorporate interest groups as a strategic actor.

This paper takes a step toward combining the two literatures – to our knowledge, the first step. We analyze an infinite horizon game in which a representative voter elects a single office-holder in each period. In each election, the voter chooses between an incumbent and a randomly-drawn challenger. The voter cares about policy outcomes, while politicians care about holding office. Each period, Nature also draws an interest group that can offer a contract to the incumbent for choosing particular policies. Each challengers’ value of office (denoted  $w$ ) is drawn from a stationary distribution and cannot be revealed to the voter until she achieves office. An incumbent’s “price” for an interest group therefore depends on

---

<sup>1</sup>See Peltzman (1976), Denzau and Munger (1986), Grossman and Helpman (1994, and 2001, Chapters 7-9), and Persson and Tabellini (2000, Chapter 7) for examples of the former, and Austen-Smith (1987), Baron (1994), and Grossman and Helpman (1996, and 2001, Chapter 10) for examples of the latter.

<sup>2</sup>A sampling of the literature includes Ferejohn (1986), Rogoff and Sibert (1988), Austen-Smith and Banks (1989), Rogoff (1990), Banks and Sundaram (1993, 1998), Harrington (1993), Fearon (1999), Barganza (2000), Besley and Smart (2001), Hindriks and Belleflamme (2001), Le Borgne and Lockwood (2001a, 2001b), Besley (2003), Smart and Sturm (2003, 2004).



$w$ . The voter thus has incentives both to retain good incumbent types as well as to induce good performance from them.

We study several variants of this model. In the first, the voter can commit to optimal stationary contracts for controlling the politician. In the second, we drop the commitment assumption and examine both stationary and simple (two-state) non-stationary equilibria. The model is then extended to incorporate finite term limits, where incumbents must be replaced after serving their  $T$ -th term. Finally, we briefly consider the cases of an infinitely-lived interest group and multiple interest groups.

Our results reveal several important features of the tension between inducing performance and selecting types. In an environment where the voter can commit to re-election contracts, she will re-elect an incumbent only if the chosen policy is sufficiently close to her ideal point. The voter may allow policy to deviate from her ideal point somewhat, however, to prevent excessive interest group vote buying. Policies that are too far from the group's ideal will induce the group to "buy" its ideal policy instead, at a cost equal to the incumbent's expected lifetime payoff. An incumbent's "price" therefore depends on her anticipation of future re-elections.

For a given incumbent type, the voter thereby maximizes her policy utility by promising re-election in all future periods. However, the voter may not wish to induce maximum performance from every incumbent type. Since incumbents who value office more highly also command higher prices, voters have an incentive to remove "low- $w$ " incumbents. Therefore, in the contracting equilibrium the voter will always keep sufficiently high quality incumbents and always remove sufficiently low quality incumbents. In between, incumbents may be retained if groups are extreme, as this allows the voter to achieve some policy performance when it is most needed. Somewhat counterintuitively, our model thus predicts that re-election rates should increase as policies diverge from the voter's ideal.

When we remove the assumption of re-election contracts, the results depend on the form of equilibrium assumed. In an equilibrium in stationary strategies, the voter's ability to induce performance is severely constrained. Since votes are cast after policies are chosen,

the voter's type-selection incentives are too strong and all incumbent types produce the same policy results as the worst type.

When we additionally drop the strong stationarity restriction, however, the results can change dramatically. We show that by using simple non-stationary "trigger" strategies the contracting equilibrium can be restored. We should note that this is not an obvious result, because we cannot use standard repeated-game punishment strategies, since the punishment instrument eliminates players.<sup>3</sup>

Finally, we consider a few extensions to this framework. When finite term limits are imposed, an incumbent's term of office acts much like her  $w$ . As an incumbent's experience increases, the price of her vote decreases. The voter therefore loses the ability to induce performance, and becomes less likely to re-elect incumbents as they become more experienced. As a result, in both the contracting and non-stationary, non-contracting environments, voters cannot benefit from term limits. We also briefly consider the effect of a long-lived interest group and multiple groups. The former weakens a voter's electoral control by giving the group more "buying power," while the latter strengthens it when groups are on ideologically opposite sides of the voter.

Several recent papers in the literature on political agency argue that elections are mainly about "selecting good types" rather than "sanctioning poor performance" (*e.g.*, Fearon, 1999; Besley and Smart, 2001; Besley, 2003). The argument is that if politicians have policy preferences, and if the game is finite or if politicians are finitely-lived, then in any subgame perfect equilibrium voters will behave as if they care only about selecting politicians with "good" preferences. Fearon (1999, page 57) states it clearly: "although the electorate would like to commit to a retrospective voting rule to motivate self-interested politicians optimally, when it comes time to vote it makes sense for the electorate to focus *completely* on the question of type: which candidate is more likely to be principled and share the public's preferences?" If voters believe the incumbent's type is better than average, then they must re-elect her, and if they believe the incumbent's type is worse than average, then they must

---

<sup>3</sup>See, *e.g.*, Dutta (1995) for a more general treatment of such repeated dynamic games.

replace her.<sup>4</sup>

This logic might apply to a world in which politicians actually have policy preferences. However, some theorists argue that, at least in the long run, office-motivated politicians will drive out policy-motivated politicians (*e.g.*, Calvert, 1986). An alternative rationale for assuming that politicians nonetheless act as if they are policy motivated is to assert that they have “induced” policy preferences – induced by some underlying contract with a set of interest groups. Our results show that the argument in the previous paragraph does not necessary hold in a world where politicians’ have “policy preferences” that are induced by payments from interest groups. As noted above, in our model the voter *can* employ a voting strategy that both sanctions and selects, even with finitely-lived politicians. All that is required is that players use non-stationary strategies of a very simple type. One key difference between a world with politicians and interest groups and a world with policy-motivated politicians is that in the first case voters must *always* monitor politicians in order to prevent bad policy outcomes, while in the second case voters who have found a “good” politician can simply let her act according to her preferences.

## 2. The Model

Our model is one of policy-making and elections in a single constituency over an infinite horizon. We denote periods with a subscript  $t$ . Players all discount future payoffs by a common factor  $\delta \in (0, 1)$ .

In each period  $t = 1, 2, \dots$  there are four players; a politician (P), a median voter (M), an interest group ( $G_t$ ), and an election challenger ( $C_t$ ). Since the incumbent is endogenous, note that P (redundantly) denotes the incumbent politician of a given period, with  $C_0$  representing the first incumbent. Thus if  $C_1$  defeats  $C_0$ , then P “becomes”  $C_1$  in period 2, and so on.

---

<sup>4</sup>Besley (2003, Chapter 3, page 14) states the problem as follows: “the existence of good types makes it impossible for voters to commit to a re-election rule as they are no longer indifferent (ex post) between voting for the incumbent and a randomly selected challenger. Voters would actually prefer to commit to the voting rule that is used under moral hazard. However, they cannot do so. This finding casts light on the fragility of the moral hazard model to a small variation of the model which includes politician types. This is because the strict indifference rule that underpins incentives in that case allows the voters to commit. Once this indifference is broken, then there is actually a constraint on optimal voting strategies.”

P and  $G_t$  care about holding office, while M and  $G_t$  care about policy. The policy in period  $t$  is an element  $x_t$  from the convex, compact set  $X \subset \mathfrak{R}$ . M and  $G_t$  can control P in different ways; the former by voting, and the latter by offering payments in return for policies chosen. It is assumed that  $G_t$  can credibly commit to these payments.

Player utilities are represented as follows. In each period  $t$ , M receives  $u^M(x_t)$ , where  $u^M : X \rightarrow \mathfrak{R}$  is continuous and single-peaked and  $m \equiv \arg \max u^M(\cdot) \in X$ . An incumbent politician  $C_\tau$  (i.e., elected in period  $\tau$ , but possibly holding office thereafter) receives a fixed benefit of  $w_\tau$  per period upon each election. We refer to  $w_t$  as a politician's "type," and use  $w$  to denote generic types. In each period  $t$ ,  $w_t$  is drawn i.i.d. according to the probability measure  $\omega : \mathcal{B}(\mathfrak{R}_+) \rightarrow [0, 1]$  with support  $\Omega$ . She does not have policy preferences, but has quasilinear utility over a non-negative transfer or "bribe"  $b_t \in B \equiv \{b : X \rightarrow \mathfrak{R}_+ \mid b \text{ measurable}\}$  offered by  $G_t$ ; thus, in period  $t$  an incumbent  $C_\tau$  receives:

$$u^P(x_t, w_\tau) = w_\tau + b_t(x_t).$$

Finally,  $G_t$  may receive non-zero utility only in period  $t$ , when she receives:

$$u^{G_t}(x_t) - b_t(x_t),$$

where  $u^{G_t} : X \rightarrow \mathfrak{R}_-$  is continuous and single-peaked. Let  $g_t \equiv \arg \max_{x_t} u^{G_t}(\cdot) \in X$  denote  $G_t$ 's ideal policy and "type." Each  $g_t$  is drawn i.i.d. according to the probability measure  $\gamma : \mathcal{B}(X) \rightarrow [0, 1]$  with support  $\Gamma$ . We assume that for all  $G_t$ ,  $u^{G_t}(x_t) = v(x_t - g_t)$  with  $v(0) = 0$ , so that group utility functions are identical up to changes in type.<sup>5</sup>

The sequence of moves in each period  $t$  is as follows. All actions are perfectly observable by all players unless otherwise noted.

*Group Draw.* Nature selects group  $G_t$ .

---

<sup>5</sup>Alternatively, we could assume that all groups have the same ideal point  $g$ , but differ in the intensity of their preferences – i.e., in their willingness to pay for favorable policy shifts. This yields qualitatively similar results.

*Vote Buying.*  $G_t$  offers  $P$  a transfer schedule  $b_t \in B$ , unobserved by  $M$ .<sup>6,7</sup>

*Policy Choice.*  $P$  chooses  $x_t \in X$ .

*Challenger Draw.* Nature selects challenger  $C_t$ , where  $w_t$  is unobserved by  $M$ .

*Election Voting.*  $M$  casts a vote  $r_t \in \{0, 1\}$  over whether to re-elect  $P$  (1) or elect a challenger  $C_t$  (0). If  $C_t$  wins,  $w_t$  is revealed to  $M$ .

We will derive subgame perfect equilibria in pure strategies. Strategies for each player are measurable mappings defined as follows. Let  $H_t$  denote the set of game histories prior to period  $t$ . Then  $G_t$ 's strategy  $\beta_t : H_t \times \Omega \times \Gamma \rightarrow B$  maps the history through period  $t-1$ , the politician type, and her type into a bribe schedule.  $P$ 's strategy  $\chi_t : H_t \times \Omega \times \Gamma \times B \rightarrow X$  maps histories, types, and bribe schedules into a policy choice. Finally,  $M$ 's strategy  $\rho_t : H_t \times \Omega \times \Gamma \times B \times X \rightarrow \{0, 1\}$  maps histories, types, bribes, and policy choices into a vote for the incumbent or challenger.

### 3. The Contracting Game

To establish a baseline for comparison, we first examine a case in which  $M$  may write “contracts.” That is, in each period  $t$ ,  $M$  commits to a vote based on actions from that period. Formally, this requires that the game be modified so that instead of choosing a vote after the policy choice,  $M$  announces  $\rho_t$  prior to  $G_t$ 's vote buying. Following convention, we restrict attention to stationary subgame perfect equilibria (SSPE). This requires that players act identically and optimally when faced with identical continuation games, and hence imply history-independent strategies.  $M$ 's period  $t$  contract can thus condition only on  $g_t$ ,  $x_t$ , and the incumbent's type. Additionally, we will focus on the set of optimal SSPE for  $M$ , which we label  $\mathcal{E}_c$ .

---

<sup>6</sup>The unobservability of the transfer schedule simplifies the analysis but is not required for our results.

<sup>7</sup>This gives all of the bargaining/agenda-setting power to the group. However, our main results and intuitions hold under a variety of different assumptions about how the bargaining/agenda-setting power is divided between the politician and the group, as discussed in note 8 below.

*A One-Shot Game.* Suppose initially that the game has only a single period. For convenience, we drop time subscripts until returning to the repeated game. M thus attempts only to induce good performance; *i.e.*,  $\rho$  is chosen to induce the optimal policy from P of type  $w$ . Because M does not care about future policy choices, the result for this case holds regardless of whether she is able to commit to  $\rho$ .

We begin by considering G's response to an arbitrary contract from M. Note first that generally, an optimal contract cannot be constant in  $x$ ; otherwise, G could achieve her ideal policy at negligible cost by offering the following contract:

$$b(x) = \begin{cases} \epsilon & \text{if } x = g \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Hence, we may partition  $X$  into two disjoint, non-empty subsets;  $\mathcal{P} = \{x \in X \mid \rho(w, g, x) = 1\}$ , and  $\mathcal{C} = \{x \in X \mid \rho(w, g, x) = 0\}$ . It is then clear that G's optimal contract must be of one of two types: (i) offer  $\epsilon$  for choosing  $\bar{p} = \arg \max_{x \in \mathcal{P}} u^G(x)$  and zero otherwise; or (ii) offer  $w + \epsilon$  for choosing  $\bar{c} = \arg \max_{x \in \mathcal{C}} u^G(x)$  and zero otherwise. G offers the type (i) contract if  $u^G(\bar{p}) > u^G(\bar{c}) - w$ .

To derive M's voting contract, let  $\mathcal{G}(y) = \{x \in X \mid u^G(x) \geq y\}$  denote the upper contour set of  $u^G(\cdot)$ . By the single-peakedness and continuity of  $u^G(\cdot)$ ,  $\mathcal{G}(y)$  is a closed interval containing  $g$ . Let  $\bar{g}(y) = \max \mathcal{G}(y)$  and  $\underline{g}(y) = \min \mathcal{G}(y)$ . There are two cases. First, if  $m \in \mathcal{G}(-w)$ , then G is unwilling to pay the cost of a type (ii) contract to move P's policy away from  $m$ . M can then threaten to re-elect P if and only if  $m$  is chosen. Second, if  $m \notin \mathcal{G}(-w)$ , then M cannot induce any policy choice outside of  $\mathcal{G}(-w)$ , for otherwise G can use a type (ii) contract to achieve  $x = g$ . Thus the best policy that M can induce is:

$$\tilde{x}(w, g) = \begin{cases} \underline{g}(-w) & \text{if } m < \underline{g}(-w) \\ m & \text{if } m \in [\underline{g}(-w), \bar{g}(-w)] \\ \bar{g}(-w) & \text{if } m > \bar{g}(-w). \end{cases} \quad (2)$$

Note that as  $w$  decreases,  $\mathcal{G}(w)$  shrinks. This reduces P's "price" for G, and in turn M's ability to discipline P.

One contract that achieves this result is:

$$\rho^*(w, g, x) = \begin{cases} 1 & \text{if } x = \tilde{x}(w, g) \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$



One optimal response for G would then be a null contract with zero payments (*i.e.*, a type (i) contract with  $\epsilon \rightarrow 0$ ). P chooses  $\tilde{x}(w, g)$  and is re-elected.<sup>8</sup>

*The Repeated Game.* In an infinite period setting, incumbents may expect higher payoffs because of multiple re-elections. As (2) suggests, this may affect the set of policies that M can induce from P. Let  $l(w; \{\rho(w, \cdot)\})$  denote a type- $w$  incumbent's *ex ante* (*i.e.*, prior to the draw of  $g_t$ ) discounted expected payoff given a set of voting contracts. We simply write  $l_w$  when the contracts in question are generic or understood. Generalizing from the above discussion, it is possible for M to induce “myopically” the following policies in period  $t$ :

$$\tilde{x}(w, g_t) = \begin{cases} \underline{g}_t(-w - \delta l_w) & \text{if } m < \underline{g}_t(-w - \delta l_w) \\ m & \text{if } m \in [\underline{g}_t(-w - \delta l_w), \bar{g}_t(-w - \delta l_w)] \\ \bar{g}_t(-w - \delta l_w) & \text{if } m > \bar{g}_t(-w - \delta l_w). \end{cases} \quad (4)$$

We extend the previous analysis to the repeated case in two steps. First, suppose that regardless of  $w$  and  $g_t$ , M offers a contract that achieves (4) in each period. Since the incumbent is always re-elected under these contracts,  $l_w = w/(1 - \delta)$ . We refer to this set of strategy profiles (which may not be Nash) as  $\mathcal{E}'_c$ .

The strategies in  $\mathcal{E}'_c$  may be thought of as a “pure” *ex post* monitoring solution, in that M’s contract achieves the best policy possible in period  $t$  without any regard for the type of politician retained. Given  $w$ , M does better here than in the single-period case because repetition automatically raises P’s continuation value.

Second, consider whether a strategy profile in  $\mathcal{E}'_c$  can be sustained as an equilibrium. There are two simple cases in which it is. If  $w_t$  is constant for all  $t$ , then M clearly has no

<sup>8</sup>Here we briefly sketch what happens when all bargaining/agenda-setting power lies with P rather than G. Consider the case with  $m < \underline{g}(-w) < g$ . Suppose that there is a status quo or reversion policy  $s \leq m$ , which is enacted if P does not propose anything or if G rejects all of P’s proposals. Suppose M offers a contract of the form:  $\rho^*(w, g, x) = 1$  if  $x \leq x_0$  and  $\rho^*(w, g, x) = 0$  otherwise, with  $x_0 < g$ . Given the “threat point”  $s$ , P can extract at most  $-u^G(s)$  in payments from G, by making G a take-it-or-leave-it offer of the form:  $x = g$  and  $b = -u^G(s)$  (which G will accept). This yields P a payoff of  $-u^G(s)$ , since P will not be re-elected under the proposed contract from M. Alternatively, P can choose a policy that allows her to be re-elected. The best such policy for P is  $x_0$  itself, together with a take-it-or-leave-it offer to G of the form:  $x = x_0$  and  $b = u^G(x_0) - u^G(s)$  (which G will again accept). This yields P a payoff of  $w + u^G(x_0) - u^G(s)$ . The optimal contract for M then solves: minimize  $x_0$  s.t.  $w + u^G(x_0) - u^G(s) \geq -u^G(s)$ , or minimize  $x_0$  s.t.  $u^G(x_0) \geq -w$ . The solution is therefore  $\tilde{x}(w, g) = \underline{g}(-w)$ . Thus, M’s optimal contract, and the equilibrium policy outcome, is exactly as in the game analyzed in the paper where G has all of the bargaining/agenda-setting power. Note also that this is true *regardless* of the reversion policy  $s$  (as long as  $s < \underline{g}(-w)$ ).

incentive to select politician types. Similarly, if  $\Gamma$  is such that  $m \in \mathcal{G}(-\frac{w_t}{1-\delta})$  for all  $g_t$  and  $w_t$ , then  $M$  can achieve her ideal policy for any politician type. In these cases, strategy profiles in  $\mathcal{E}'_c$  are also an optimal equilibria, so  $\mathcal{E}_c = \mathcal{E}'_c$ .

More generally, in an optimal SSPE,  $M$  will always replace an incumbent if better performance is expected from a replacement in the long run. Let  $v_c(w) = \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E} u^M(x_t^* \mid w_0 = w)$  represent  $M$ 's discounted expected utility from a type- $w$  incumbent in the repeated contracting game, where  $x_t^*$  is the policy choice under optimal play. Clearly, if  $M$  does not re-elect  $P$ , then  $G$  can offer a contract similar to (1), resulting in policy  $g_t$ . Thus,  $M$  re-elects  $P$  if:

$$u^M(\tilde{x}(w, g_t)) + \delta v_c(w) \geq u^M(g_t) + \delta \int_{\Omega} v_c(\tilde{w}) \omega(d\tilde{w}). \quad (5)$$

Note that because a type- $w$  incumbent's future election prospects may depend on future realizations of  $g_t$ , letting  $\epsilon \rightarrow 0$ , her expected payoff from re-election must fall within  $[w, \frac{w}{1-\delta}]$ . Hence  $\tilde{x}(w, g_t)$  lies between the solutions of (2) and (4) under contracts in  $\mathcal{E}'_c$ . Thus,  $u^M(\tilde{x}_t) \geq u^M(g_t)$ , with the inequality strict for all  $x_t \neq g_t$ .

The integral in (5) represents the discounted expected value from electing a challenger  $C_t$ , prior to the revelation of  $w_t$ . Obviously, it provides an important point of comparison for  $M$  in her election choice. It will therefore be convenient to define an ‘‘average’’ type  $\hat{w} = \inf \{w \mid v_c(w) \geq \int_{\Omega} v_c(\tilde{w}) \omega(d\tilde{w})\}$ , which is the lowest type that is *ex ante* at least as good as a challenger. We do not require that  $\hat{w} \in \Omega$ ; rather, it may be thought of as a one-time incumbent in a fictional game in which future challengers are all drawn according to  $\omega$ .

Our first result uses these observations to characterize some of the policy and re-election implications of optimal equilibria. It establishes a basic monotonicity of re-election results according to type: all incumbents with types above  $\hat{w}$  are re-elected. Those below may not be re-elected for certain group types (*i.e.*, they do not satisfy (5)). In the latter case, if  $u^M$  is concave, then somewhat counter-intuitively, incumbents are increasingly re-elected when groups (and hence equilibrium policies) are *farther* from  $m$ . Finally, sufficiently low incumbent types are never re-elected.

**Proposition 1** *In any equilibrium in  $\mathcal{E}_c$ , there exist  $w^1$  and  $w^2$  such that  $0 \leq w^1 \leq w^2 \leq \hat{w}$  and:*

$$r_t^* = \begin{cases} 0 & \text{if } w < w^1 \\ \eta(w, g_t) & \text{if } w \in [w^1, w^2] \\ 1 & \text{if } w > w^2, \end{cases}$$

*for some  $\eta : [w^1, w^2] \times \Gamma \rightarrow \{0, 1\}$ .  $\eta(w, g_t)$  is non-decreasing in  $w$ , and non-decreasing in  $|m - g_t|$  if  $u^M$  is concave. ■*

**Proof.** All proofs are in the Appendix.

A few observations about Proposition 1 are worth noting. First, while the result is stated for  $\mathcal{E}_c$ , it also holds for many (but not necessarily all) SSPEs. Second, neither  $w^1$  nor  $w^2$  need belong to  $\Omega$ ; they are simply cutpoints characterizing behavior for any  $w$ . Third, re-election outcomes depend greatly on  $\gamma$ . For example, if  $\gamma$  is degenerate with  $g_t \neq m$ , then  $w^1 = w^2$ . And if some neighborhood of  $m$  is not contained within  $\Gamma$  and  $\Omega = \mathbb{R}_+$ , then some “slightly” below average types may always be re-elected. Finally, the effect of  $\delta$  is ambiguous. High values can enlarge  $\mathcal{G}_t(\cdot)$  and hence the range of group types for which M can achieve her ideal policy, *holding re-election strategies constant*. However, they also give M a greater incentive to select good future politicians, thus expanding the set of politician and group types for which incumbents are kicked out.

The exercise here establishes that in an environment in which voters may write “contracts” for politicians, an optimal strategy in a repeated game solves both shirking and type-selection problems. But because of the bluntness of the voter’s contract instrument, both problems are solved in crude fashion. The voter may only select against the worst politician types (and only under certain circumstances), and may only move policy a limited amount when groups are extreme.

## 4. Equilibria in Stationary Strategies

We now return to the case in which M cannot write contracts, but instead votes in a sequentially rational manner. We again restrict attention to SSPE, and focus on the optimal such equilibria for M, which we label  $\mathcal{E}_s$ .

Analogously with the previous section, we define  $v_s(w)$  to be M's expected payoff in an arbitrary SSPE, starting from a period with a type- $w$  incumbent. At the voting stage, M's problem is to choose between a type- $w$  incumbent and a random draw from among the challengers. By stationarity, she re-elects P if:

$$v_s(w) \geq \int_{\Omega} v_s(\bar{w})\omega(d\bar{w}). \quad (6)$$

This implies that if P is “above average,” then she is always retained. Likewise, if P is “below average,” she is never re-elected. As a result, for all types such that (6) is not satisfied with equality,  $\rho_t^*(\cdot)$  must be constant in  $x_t$ . In response,  $G_t$  can offer a contract of the form in (1), and thus receives her ideal policy in every such period. The following result uses this intuition to establish that all continuation values (up to a set of measure zero) must be equal.

**Comment 1** *In any SSPE,  $v_s(w) = k$  almost everywhere for some  $k$ . ■*

Thus, in equilibrium M's incentive to select types effectively prevents monitoring of performance. Whereas in the contracting case M could induce high- $w$  types to choose “better” policies, she cannot credibly force different types to choose different actions in a stationary equilibrium.

There are many SSPEs. As a trivial example, consider the equilibrium set  $\mathcal{E}_0$ , in which  $G_t$  offers a contract of the form in (1) in each period. P chooses  $g_t$ , and M elects  $C_t$ . Note that since  $G_t$  always receives her ideal policy,  $v_s(w)$  is constant. Clearly, this is the worst equilibrium for M. A more plausible equilibrium might feature the optimal (credible) use of re-election incentives.

To characterize  $\mathcal{E}_s$ , we make use of Comment 1. Denote by  $\underline{w} \equiv \inf\{\bar{w} \in \Omega\}$  the “lowest” candidate type. If  $w = \underline{w}$ , then (similar to the discussion of the contracting case) with the appropriate voting strategy, M can induce any policy in  $\mathcal{G}_t(-\frac{\underline{w}}{1-\delta})$  in each period  $t$ . At best, it can then achieve a policy of  $m$  if  $m \in \mathcal{G}_t(-\frac{\underline{w}}{1-\delta})$ , and  $\underline{g}_t(-\frac{\underline{w}}{1-\delta})$  or  $\bar{g}_t(-\frac{\underline{w}}{1-\delta})$  otherwise. Thus, if  $\underline{w}$  were the *only* type in  $\Omega$ , then P is indifferent between all candidates and can thereby achieve the contracting result.

The following result establishes that while M can credibly induce any type of politician to take the same action as type- $\underline{w}$  (hence creating a uniform expected value for all types), she cannot do any better. The reason for this is straightforward: if M could induce a better policy in expectation, then this policy must sometimes lie outside of  $\mathcal{G}_t(-\frac{w}{1-\delta})$ . In these cases,  $G_t$  would then be willing to “buy out” type- $\underline{w}$  politicians by offering  $\frac{w}{1-\delta} + \epsilon$  for a policy choice of  $g_t$ .

**Proposition 2** *In any equilibrium in  $\mathcal{E}_s$ , for any type- $w$  incumbent and all  $t$ ,  $g_t$ , and  $b_t$ :  $\rho_t^*(w, g_t, b_t, \chi_t^*) = 1$  and*

$$\chi_t^*(w, g_t, b_t) = \begin{cases} \underline{g}_t(-\frac{w}{1-\delta}) & \text{if } m < \underline{g}_t(-\frac{w}{1-\delta}) \\ m & \text{if } m \in [\underline{g}_t(-\frac{w}{1-\delta}), \bar{g}_t(-\frac{w}{1-\delta})] \\ \bar{g}_t(-\frac{w}{1-\delta}) & \text{if } m > \bar{g}_t(-\frac{w}{1-\delta}). \quad \blacksquare \end{cases}$$

As in the contracting case, M’s strategy has elements of type selection and *ex post* monitoring. However, removing commitment to re-election schedules makes the selection of types irrelevant in equilibrium. This happens because of stationarity, as well as the fact that M’s election choice occurs after P’s policy choice. The incentive to retain high types (respectively, remove low types) is then so strong that if they existed, they would always be re-elected (respectively, removed). Under these conditions  $G_t$  can easily buy its ideal policy. Thus the best that M can do is to monitor all types uniformly, and in a credible fashion. This causes the performance of each type to sink to that of the “least common denominator,” or  $\underline{w}$ , regardless of the distribution of the rest of candidate types.

## 5. Non-Stationary Strategies

The preceding results suggest that the ability to commit might play a central role in voter welfare. Here we demonstrate that this is not so. Instead, by dropping the requirement of stationary equilibria and focusing instead on simple, non-stationary equilibria, the voter can do just as well as she could in the contracting case.

We begin by noting a major difference between our model and other dynamic games in which non-stationary solution concepts are applied. In the game examined here, the only



plausible “punishment” device (*i.e.*, not re-electing P) removes the punished player. That is, we do not have a repeated game. Combined with complete information, this greatly simplifies a potential defector’s optimization problem. It also complicates the potential punisher’s problem, in that it limits the promise of future “cooperative” interaction.<sup>9</sup>

There are many kinds of non-stationary equilibria. For example, the voter might punish a politician by not re-electing him, and “cooperate” with new politicians at a later date. It may also condition on the type of group drawn. However, our results require only that we examine the simplest such class of equilibria, in which P and  $G_t$  play a “trigger” strategy against M. Such equilibria are characterized by the triple  $(\mathcal{E}^C, \mathcal{E}_0, n)$ , the elements of which correspond to a *cooperative* phase, *punishment* phase, and punishment length, respectively. Play begins in the cooperative phase at  $t = 1$ , and upon any deviation by M from its prescribed strategies, play continues in the punishment phase for  $n \geq 1$  periods. During this phase,  $G_t$  offers a contract of the form in (1), P chooses  $g_t$  in each period, and M never re-elects P. After the punishment phase, play reverts to the cooperative phase. Subgame perfection requires that the game play within the punishment phase itself be a subgame perfect Nash equilibrium, and that the strategies in the cooperative phase be consistent with the incentives posed by the punishment phase.

Cooperative phase strategies may take many forms, and for simplicity we focus on strategies in  $\mathcal{E}^C$  that meet two criteria. First, they must be stationary.<sup>10</sup> Second, they cannot result in the punishment phase along the equilibrium path (though the punishment phase may be essential for inducing M to play according to  $\mathcal{E}^C$ ). Clearly, then, the punishment phase cannot be reached in equilibrium if the cooperative phase strategies yield a policy at least as good as  $g_t$  in each period, and  $n$  is sufficiently large. We call strategies satisfying these requirements *stationary trigger* strategies.

Even with its restrictions, many stationary trigger equilibria exist. The simplest example might be a “grim trigger” equilibrium  $(\mathcal{E}_s, \mathcal{E}_0, \infty)$ , where play proceeds according to the

---

<sup>9</sup>See Dutta (1995) for a more general treatment of equilibria in dynamic games.

<sup>10</sup>Note that the punishment phase strategies are stationary as well.



stationary equilibrium of the previous section. Another, more plausible, example might be  $(\mathcal{E}'_c, \mathcal{E}_0, \infty)$ , where M votes as she would in the “myopic” contracting game. M has no incentive to deviate from its specified voting strategy, because doing so will result in receiving  $g_t$  forever, a result which is weakly dominated by that under  $\mathcal{E}^C$ . Clearly, then, there is room for discrimination amongst types even in a very rudimentary non-stationary setup.

As both of these examples illustrate, M’s minmax payoff is simply that implied by  $\mathcal{E}_0$ ; *i.e.*, receiving a policy at  $g_t$  in every period. Moreover, M’s vote is the last action in each period, and does not affect her payoffs in that period.<sup>11</sup> However, M may still exploit P by replacing a “bad” incumbent after she chooses a policy in expectation of re-election. Thus, to establish a trigger equilibrium it will be important to check that M does not have sufficient incentive to do so.

The following result uses these observations to characterize the optimal equilibrium for M, which restores the contracting outcome  $\mathcal{E}_c$  as the predicted result.<sup>12</sup>

**Proposition 3** *For  $n$  sufficiently large, in the optimal stationary trigger strategy equilibrium,  $\mathcal{E}^C = \mathcal{E}_c$ . ■*

As intuition might suggest, sufficiently long (and possibly infinite) punishment periods will ensure that M does not exploit weak incumbents. Because there is no noise in the observables, the punishment phase is never invoked in equilibrium. Thus, the simple non-stationary equilibrium derived in Proposition 3 produces policies that are identical in every period to those of the contracting equilibrium of Proposition 1.

*Example 1.* M and  $G_t$  have quadratic utility, with  $u^M(x) = -x^2$  and  $u^{G_t}(x) = -(g_t - x)^2$ . Group and candidate types are distributed as follows:  $\Gamma \equiv \{1, 8\}$ ,  $\gamma(1) = \gamma(8) = 0.5$ , and  $\Omega \equiv \{1, 5\}$ ,  $\omega(1) = \omega(5) = 0.5$ . The discount factor for all players is  $\delta = 0.8$ .

It is clear that  $1 < \hat{w} < 5$ . A type-5 incumbent must always be re-elected, and thus its expected lifetime payoff is  $5/(1-\delta) = 25$ . This implies that M can induce a type-5 incumbent

<sup>11</sup>The latter point implies that there is no analog to “suckering” one’s opponent in the prisoner’s dilemma.

<sup>12</sup>The proof is easily modified to accommodate non-optimal stationary trigger equilibria as well.

to choose a policy up to 5 units away from  $g_t$ . Thus, when  $g_t = 1$  (8), the policy is 0 (3), and  $v_c(5) = -4.5/(1-\delta) = -22.5$ . Suppose that a type-1 incumbent is only re-elected if  $g_t = 8$ ; this results in an expected lifetime payoff of  $1(0.5)/(1-\delta(0.5)) = 5/6$ . Thus, when  $g_t = 1$  (8), the policy is 1 ( $\approx 6.71$ ), and  $v_c(1) \approx 0.5[-1+\delta(v_c(1)+v_c(5))]/2 + 0.5[-6.71^2+\delta v_c(1)] \approx -68.76$ . A similar exercise reveals that M would receive  $-83.06$  from always re-electing a type-1 incumbent,  $-69.17$  from never re-electing a type-1 incumbent, and  $-91.25$  from only re-electing when  $g_t = 1$ ; thus the strategy of re-electing only when  $g_t = 8$  is optimal in a contracting environment.

To complete the stationary trigger equilibrium, we check that M will follow its prescribed strategy. It is obvious that for any  $n$ , M will always have sufficient incentive to re-elect a type-5 incumbent. Similarly, M will clearly not re-elect a type-1 incumbent when  $g_t = 1$ . Finally, M re-elects a type-1 incumbent when  $g_t = 10$  if:

$$v_c(1) \geq \sum_{j=0}^{n-1} \delta^j Eu^M(g_t) + \delta^n \frac{v_c(1) + v_c(5)}{2}. \quad (7)$$

Substituting in values from above, (7) holds for any  $n \geq 1$ . Thus even a mild punishment scheme is capable of sustaining “contract” behavior in this example.

## 6. Extensions

### 6.1 Term Limits

We now extend our results to the case in which legislators may serve no more than  $T > 0$  terms. The model is unchanged with the exception that if, in period  $t$ , an incumbent completes her  $T$ -th period of office, she is automatically replaced with the challenger  $C_t$ . Note that at  $T = \infty$ , the model is identical to that in the previous sections. We continue to assume that a legislator who leaves office cannot return as a candidate.

The main intuition of our results is that an incumbent’s continuation value will depend not only on her value of holding office, but also on the number of possible terms remaining. Thus the extent to which M can control politicians will also depend on both variables. Let  $\theta \in \{1, \dots, T\}$  denote the term that the legislator is currently serving.

We begin with the optimal contracting equilibria, which we label  $\mathcal{E}_T$ . Let  $v_T(w, \theta)$  represent the expected value to M of a type- $w$  legislator with  $\theta$  terms of experience. As in Section 3, M has two choices at the contracting stage. First, she may extract the optimal performance given P's expected lifetime and re-elect the incumbent, who will be of type  $(w, \theta+1)$  in the subsequent period. Second, M may allow  $G_t$  to buy its ideal policy and elect a type- $(w_t, 1)$  challenger, where  $w_t$  is randomly drawn as before.

To reflect the possible dependence of strategies on  $\theta$ , we extend the notation of Section 3 as follows. Let  $l_{w,\theta}$  denote the discounted expected payoffs of a type- $(w, \theta)$  incumbent prior to the draw of  $g_t$ . Next, let  $\tilde{x}(w, \theta, g_t)$  represent the optimal policy that M can induce through its re-election contract (of the form in (3)) in a given period:

$$\tilde{x}(w, \theta, g_t) = \begin{cases} \underline{g}_t(-w - \delta l_{w,\theta}) & \text{if } m < \underline{g}_t(-w - \delta l_{w,\theta}) \\ m & \text{if } m \in [\underline{g}_t(-w - \delta l_{w,\theta}), \bar{g}_t(-w - \delta l_{w,\theta})] \\ \bar{g}_t(-w - \delta l_{w,\theta}) & \text{if } m > \bar{g}_t(-w - \delta l_{w,\theta}). \end{cases} \quad (8)$$

Thus, the optimal contract re-elects P if:

$$u^M(\tilde{x}(w, \theta, g_t)) + \delta v_T(w, \theta+1) \geq u^M(g_t) + \delta \int_{\Omega} v_T(\tilde{w}, 1) \omega(d\tilde{w}). \quad (9)$$

Finite term limits allow us to pin down the expected payoff from an incumbent in her final term ( $\theta = T$ ). Since she cannot be re-elected, no voting contract can induce any performance. By offering a contract of the form in (1),  $G_t$  can then obtain its optimal policy (*i.e.*,  $\tilde{x}(w, T, g_t) = g_t$ ). Thus,

$$v_T(w, T) = \int_{\Gamma} u^M(g) \gamma(dg) + \delta \int_{\Omega} v_T(\tilde{w}, 1) \omega(d\tilde{w}). \quad (10)$$

Expression (10) also implies that legislators in their penultimate terms will also be difficult to control. With a single possible period of office remaining at period  $t$ , M can extract some performance by promising a single re-election. However, doing so will result in a period  $t+1$  policy of  $g_{t+1}$ , followed by the election of challenger  $C_{t+1}$ . Unless future payoffs are discounted heavily, or  $G_t$  is extreme and M's utility is concave, M may simply prefer electing challenger  $C_t$  to re-electing P.

This logic suggests that  $M$  has a greater ability to use future elections to discipline incumbents earlier in their careers. Loosely speaking, low values of  $\theta$  have an effect similar to that of high values of  $w$ . Term limits therefore constrain the inducements that may be offered even to the “best” incumbents, and reduce the long-term expected quality of policies (from  $M$ ’s perspective). In an environment with interest groups and voting contracts, voters would then not be able to benefit from finite term limits.<sup>13</sup> The following result establishes these intuitions.

**Proposition 4** (i)  $v_T(w, \theta)$  is non-decreasing in  $w$ .

(ii)  $v_T(w, \theta)$  is non-increasing in  $\theta$ .

(iii)  $v_T(w, \theta) \leq v_c(w)$ ,  $\forall T, \theta, w \in \Omega$ . ■

Combined with (9), an easily established corollary is that  $M$ ’s desire for policy performance becomes more demanding as  $\theta$  increases or  $w$  decreases. As with the non-term limited model, this implies that if  $u^M(\cdot)$  is concave, then progressively worse draws of  $g_t$  are required to keep an incumbent in office.

Since incumbents become less desirable with seniority, our model suggests that when term limits are finite, re-election rates should change over time. Previous models of term limits that do not incorporate citizens’ type-selection incentives (*e.g.*, Lopez 2002) might therefore underestimate the impact of term limits on incumbent turnover rates.

The next result shows that an incumbent’s *ex ante* re-election probability, or the probability re-election prior to the draw of  $G_t$ , decreases weakly over time.<sup>14</sup>

**Proposition 5** The *ex ante* re-election probability of any incumbent is non-increasing in  $\theta$ .

■

Because type- $(w, T)$  incumbents have an *ex ante* re-election probability of zero, Proposition 5 implies that unless incumbents are never re-elected (*e.g.*, if  $\omega(0) = 1$ ,  $\gamma(m) = 1$ , or

<sup>13</sup>In the model of Smart and Sturm (2004), term-limits can help voters, but no interest groups are present.

<sup>14</sup>Consistent with this prediction, Smart and Sturm (2004) estimate that term-limited U.S. governors are less likely to be re-elected than governors without term limits.

$T = 1$ ), there exist incumbent types for which the *ex ante* re-election probability is strictly decreasing. The result also compares usefully with Proposition 1, which established that some types ( $w > \hat{w}$ ) are always retained. With finite term limits, a type- $(w, \theta)$  incumbent might be retained with certainty for sufficiently high  $w$  and low  $\theta$ , but her *ex ante* re-election probability declines to zero eventually.

Our final result eliminates the assumption of voting contracts and is therefore an analog to Proposition 3. We consider the same class of stationary trigger strategies as in the previous section. As with the non-term limited case, the result is that simple non-stationary strategies (such as  $(\mathcal{E}_T, \mathcal{E}_0, \infty)$ ) are sufficient to support strategy profiles in  $\mathcal{E}_T$ .

**Proposition 6** *For  $T$  finite and  $n$  sufficiently large, in the optimal stationary trigger strategy equilibrium,  $\mathcal{E}^C = \mathcal{E}_T$ . ■*

*Example 2.* We maintain all of the parameters of Example 1, and additionally assume  $T = 2$ . Clearly, the expected lifetime payoff of  $T$ -th term incumbents is zero for all  $w$ , and any re-elected politician will be induced by  $G_t$  to choose a policy of  $g_t$ . The expected policy utility from  $g_t$  is  $-32.5$ ; thus,  $v_T(5, 2) = v_T(1, 2) = \bar{v}_2 = -32.5 + \delta(v_T(1, 1) + v_T(5, 1))/2$ .

Now consider the set of possible contracts for newly-elected politicians. If  $M$  does not re-elect, then it will receive  $g_t$ , while by re-electing a type- $(w, 1)$  incumbent will expect a lifetime payoff of  $w$ . As a result,  $M$  can induce a policy of 0 for  $g_t = 1$ , and a policy of  $\approx 5.76$  (7) for  $g_t = 8$  and  $w = 5$  (1). With these values, it is possible to calculate the expected values of each possible contract. We provide several examples here and omit several obviously non-optimal cases:

- Never Re-elect:  $v_T(w, 1) = \bar{v}_2$  for all  $w$ ;  $v_T(w, 1) = -162.5$ .
- Re-elect iff  $w = 5$ :  $v_T(1, 1) = \bar{v}_2$  and  $v_T(5, 1) \approx -5.76^2/2 + \delta\bar{v}_2$ ;  $v_T(1, 1) \approx -139.80$ ,  $v_T(5, 1) \approx -128.45$ .
- Always Re-elect:  $v_T(1, 1) \approx -7^2/2 + \delta\bar{v}_2$  and  $v_T(5, 1) \approx -5.76^2/2 + \delta\bar{v}_2$ ;  $v_T(1, 1) \approx -133.27$ ,  $v_T(5, 1) \approx -125.38$ .

- Re-elect iff  $w = 5$  and  $g_t = 8$ :  $v_T(1, 1) = \bar{v}_2$  and  $v_T(5, 1) \approx (-5.76^2 + \delta\bar{v}_2)/2 + [-1^2 + \delta(v_T(1, 1) + v_T(5, 1))/2]/2$ ;  $v_T(1, 1) \approx -136.85$ ,  $v_T(5, 1) \approx -124.03$ .
- Re-elect iff  $w = 5$  or  $g_t = 8$ :  $v_T(1, 1) = (-7^2 + \delta\bar{v}_2)/2 + [-1^2 + \delta(v_T(1, 1) + v_T(5, 1))/2]/2$  and  $v_T(5, 1) \approx -5.76^2/2 + \delta\bar{v}_2$ ;  $v_T(1, 1) \approx -128.69$ ,  $v_T(5, 1) \approx -123.22$ .
- Re-elect iff  $g_t = 8$ :  $v_T(1, 1) = (-7^2 + \delta\bar{v}_2)/2 + [-1^2 + \delta(v_T(1, 1) + v_T(5, 1))/2]/2$  and  $v_T(5, 1) \approx (-5.76^2 + \delta\bar{v}_2)/2 + [-1^2 + \delta(v_T(1, 1) + v_T(5, 1))/2]/2$ ;  $v_T(1, 1) \approx -125.57$ ,  $v_T(5, 1) \approx -117.68$ .

Thus, M would ideally write contracts that promise new politicians re-election when groups are extreme. This strategy minimizes the damage that extreme ( $g_t = 8$ ) groups can do, and is not very costly because the policies “bought” by moderate groups ( $g_t = 1$ ) is relatively benign.

It is finally necessary to check that M does not have an incentive to deviate from its re-election strategy. Carrying a calculation identical to that in (7), M will re-elect an incumbent if  $n \geq 2$ .

## 6.2 Infinitely-Lived Groups

Now consider the case of a single group, G, that is present in all periods.<sup>15</sup> The preceding analysis was simplified by the fact that groups did not care about incumbents’ electoral fates. Under repeated interaction, however, the group also participates in the retention of incumbents.

To see how an infinitely-lived group may affect voting and bribing strategies, suppose that a voter is able to offer optimal contracts, as in  $\mathcal{E}_c$ . Under its (myopic) bribing strategy, G must accept a relatively unfavorable policy in all periods once M finds a sufficiently good politician type. G does better in periods when M chooses not to re-elect the incumbent, as the policy will be her ideal point,  $g$ . It is clear, then, that G has an incentive to “buy out”

---

<sup>15</sup>For a treatment of dynamic interaction of long-lived interests and politicians in the regulatory context, see Salant (1995), Martimort (1999), and Faure-Grimaud and Martimort (2003).



high incumbent types in order to induce a fresh draw of politicians. In turn, M will have an incentive to loosen its contract to prevent G from forcing such replacements.

We illustrate this logic with two-type example, in which we solve for the analog of  $\mathcal{E}_c$ ; that is, stationary contracts that are optimal for M.

*Example 3.* As in the previous examples, let  $u^M(x) = -x^2$ ,  $\Omega \equiv \{1, 5\}$ ,  $\omega(1) = \omega(5) = 0.5$ , and  $\delta = 0.8$ . G's utility is now given by  $u^G(x) = -(8 - x)^2$ . Under  $\mathcal{E}_c$ , M always retains type-5 incumbents, and always removes type-1 incumbents. This results in a policy at 3 (8) under type-5 (type-1) incumbent. It is easily calculated that  $v_c(5) = -45$  and  $v_c(1) = -64 + \delta(v_c(1) - 45)/2 \approx -136.67$ .

Under the strategies in  $\mathcal{E}_c$ , G would have a continuation value of  $v_c^G(5) = -(8 - 3)^2/(1 - 0.8) = -125$  for a type-5 incumbent, and  $v_c^G(1) = 0 + \delta(v_c^G(1) - 125)/2$ , implying  $v_c^G(1) \approx -104.17$  for a type-1 incumbent. G can do better by paying  $25 + \epsilon$  to a type-5 incumbent to choose a policy at  $g = 8$ . This results in a new incumbent, and an expected payoff of  $-25 - \epsilon + \delta(-104.17 - 125)/2 \approx -116.67$ .

To induce G not to buy out a type-5 incumbent, G's continuation value from a type-5 incumbent must solve the following system:

$$\begin{aligned} v^G(1) &= \frac{\delta}{2} (v^G(5) + v^G(1)) \\ v^G(5) &= -25 + \frac{\delta}{2} (v^G(5) + v^G(1)). \end{aligned}$$

Solving yields  $v^G(1) = -50$  and  $v^G(5) = -75$ . Since a type-5 incumbent is always re-elected, this implies that its policy choice  $x$  is given by  $-\frac{(8-x)^2}{1-\delta} = -75$ , or  $8 - \sqrt{15}$ , which is more distant from M's ideal than the policy (3) chosen by a short-lived group. M's optimal contract therefore re-elects a type-5 incumbent if she chooses any policy in  $[-8 + \sqrt{15}, 8 - \sqrt{15}]$ . G's best response is then to offer some  $\epsilon > 0$  for choosing policy at  $8 - \sqrt{15}$ . The resulting policy choices reduce M's expected payoff: now  $v_c(5) = 80\sqrt{15} - 395 \approx -85.16$  and  $v_c(1) \approx -64 + \delta(v_c(1) - 85.16)/2 \approx -163.44$ .

As the example suggests, we claim that optimal contracting equilibria of games with

larger type sets will share two features. First, because there is no variation in group ideal points, M will retain all sufficiently high types, and kick out all lower types. Second, for any type that is always retained by M, G will be indifferent between paying  $w/(1-\delta)$  to buy out P and allowing her to remain in office. This implies that higher types will continue to choose policies that are closer to M's ideal, but that these policies will be closer to  $g$  than under  $\mathcal{E}_c$ .

### 6.3 Two Interest Groups

The final extension considers what happens when two groups are drawn in each period, who simultaneously offer voting contracts to P. The groups are generically labeled  $G^i$ , with ideal points  $g^i$  (with positive support on  $\Gamma^i$ ), utility functions  $u^{G^i}(\cdot)$ , and upper contour sets  $\mathcal{G}^i(\cdot)$  ( $i = 1, 2$ ). As in the basic model, each group “lives” for a single period. Now each group's bribes must take into account not only the cost of the politician's expected remaining payoff from office, but also any bribes from the other group as well.

The effect of an additional group is again usefully illustrated by supposing that a voter offers optimal contracts according to  $\mathcal{E}_c$ , to group  $G^1$ . In the presence of only  $G^1$  (with  $g^1 > m$ ) and an incumbent of type  $w$ , this induces some policy  $x' \in [m, g^1]$ . But if  $x' \notin \mathcal{G}^2(-\frac{w}{1-\delta})$ , then  $G^2$  will have an incentive to “buy out” P. This upsets the bribing strategy of  $G^1$ , and raises the possibility that  $G^1$  might in turn be willing to pay more to buy back P. M may also have an incentive to adjust its contract to prevent policy from being drawn too far away from  $m$ .

The fully-developed model of competitive vote-buying is quite complex, so we simply illustrate two general features here. In both examples, we show that under a variety of realized distributions of  $g^i$ , the median voter can achieve her ideal policy with a very simple stationary voting contract.

*Example 4.* Let  $m \in \mathcal{G}^1(-\frac{w}{1-\delta}) \cap \mathcal{G}^2(-\frac{w}{1-\delta})$  for all  $(g^1, g^2) \in \Gamma^1 \times \Gamma^2$ . Suppose that P anticipates re-election in all future periods, and that M adopts the following contract: re-elect P if and only if she chooses  $m$ . Then if  $G^1$  offers no payment to P for any policy,  $G^2$ 's best response is identical to the one-group case; *i.e.*, offer no payment for any policy.

By symmetry,  $G^1$ 's best response is to offer no payment for any policy. P therefore chooses policy  $m$ . Since this outcome is clearly optimal for M for any draw of  $(g^1, g^2)$ , M always re-elects P; hence P's expected payoff at the beginning of each period is  $\frac{w}{1-\delta}$ . Thus, the contract to re-elect a type- $w$  incumbent if and only if P chooses  $m$  is part of a subgame perfect equilibrium.

*Example 5.* Let  $u^{G^i}(\cdot)$  be concave and symmetric,  $g^1 < m < g^2$ ,  $m - g^1 > g^2 - m$ , and  $m \notin G^1(-\frac{w}{1-\delta})$  for all  $(g^1, g^2) \in \Gamma^1 \times \Gamma^2$ . Thus, without loss of generality,  $G^1$  would always wish to buy out P in the absence of a bribe from  $G^2$ .

Suppose that P anticipates re-election in all future periods, and that M adopts the contract: re-elect P if and only if she chooses  $m$ . Further, let  $G^1$  offer the contract:

$$b^1(x) = \begin{cases} -u^{G^1}(g^2) & \text{if } x = g^1 \\ u^{G^1}(m) - u^{G^1}(g^2) & \text{if } x = m \\ 0 & \text{otherwise.} \end{cases}$$

Likewise, let  $G^2$  offer the contract:

$$b^2(x) = \begin{cases} -u^{G^2}(g^1) & \text{if } x = g^2 \\ -w/(1-\delta) - u^{G^2}(m) + \epsilon & \text{if } x = m \\ 0 & \text{otherwise.} \end{cases}$$

Under these strategies, P receives  $-u^{G^2}(g^1) = -u^{G^1}(g^2)$  for choosing  $g^1$  or  $g^2$ ,  $-u^{G^2}(g^1) + \epsilon$  for choosing  $m$ , and 0 otherwise. P then chooses  $m$ . For each group  $G^i$ , at least  $-u^{G^2}(g^1)$  must be offered to P to choose any policy other than  $m$  (otherwise, P will strictly prefer choosing  $g^{-i}$ ), so  $G^i$  can receive no more than  $-u^{G^2}(g^1)$  for any  $x \neq m$ . Under  $b^1(x)$  and  $b^2(x)$ , however,  $G^1$  receives  $-u^{G^2}(g^1)$  and  $G^2$  receives  $w/(1-\delta) + 2u^{G^2}(m) - \epsilon$ , which (by concavity) is strictly better. Thus, letting  $\epsilon \rightarrow 0$ ,  $b^1(x)$  and  $b^2(x)$  are best responses to M's voting contract. Since this outcome is clearly optimal for M for any draw of  $(g^1, g^2)$ , M always re-elects P; hence P's expected payoff is  $\frac{w}{1-\delta}$ . This set of bribing and voting contracts is therefore part of a subgame perfect equilibrium.

The two examples may be combined straightforwardly to show that when group utility functions are symmetric and concave, and groups are either "close" to  $m$  (in the sense that

$m \in \mathcal{G}^1(-\frac{w}{1-\delta}) \cap \mathcal{G}^2(-\frac{w}{1-\delta})$  or on opposite sides of  $m$ , then  $M$  can always obtain a policy at  $m$  for a type- $w$  incumbent. This incumbent is always re-elected.

These examples suggest that multiple groups can play a significant role in moderating policy outcomes. Extreme policy outcomes may result if both groups are on the same side of  $m$ , and at least one group is not close. It seems safe to conjecture that the outcome will never be more extreme than the most extreme group. However, since the most extreme group will tend to be more extreme as the number of groups increases, the overall effect of adding groups on the same side of  $m$  is unclear. We leave this for future work.

## 7. Discussion

The models developed here integrate strategic interest groups and strategic voters in a general framework of policy-making and elections. This combination introduces a tension in a voter's incentives, because there may be a strong short-run incentive to use electoral discipline to improve policy, and because a group can exploit a promise to retain a politician. As a result, the strategies of selecting good politician types and rewarding good performance are often incompatible.

The results illustrate the effects of this tension under a variety of assumptions. The models generally predict that incumbents who value office highly are difficult to discipline. Incumbents who value office less highly can be disciplined as the interest group becomes more extreme, because the short-term gain from policy performance is greater. This causes re-election rates to increase when adopted policies are more extreme. We also find that a long-lived interest group weakens a voter's electoral control, while multiple groups may strengthen it.

The extension to term-limited officials predicts that term limits have an effect similar to a reduction in the value of holding office. Thus, term limits reduce policy performance from a voter's perspective. In turn, they give voters a greater incentive to replace officials as their limit approaches. Our model cannot explain the apparently high level of support for term limits among citizens. While it is predicted that most voters should oppose them,

in survey and initiative voting data voters typically support term limits by a 2 to 1 margin. On the other hand, one key prediction of our model *is* consistent with the data. Specifically, there is considerable evidence that conservative and Republican voters are more supportive of term limits than liberal and Democratic voters. The policy preferences of conservative voters are probably more closely aligned with organized interest groups, since most well-organized interest groups have a pro-business, conservative in orientation (*e.g.*, Schlozman and Tierney, 1986). Term limits are therefore likely to move policy in their direction.

## Appendix

**Proof of Proposition 1.** For convenience we use  $w$  to denote the type of a generic incumbent. By stationarity, we also drop (wlog) all time subscripts.

We first define an extended value function  $\tilde{v}_c : \mathfrak{R}_+ \rightarrow \mathfrak{R}$ , which satisfies: (i)  $\tilde{v}_c(w) = v_c(w)$  for any  $w \in \Omega$ , and (ii)  $\tilde{v}_c(w) = \sum_{t=1}^{\infty} \delta^{t-1} \mathbb{E}[u^M(x^* \mid w_0 = w)]$  for any  $w \notin \Omega$ . That is,  $\tilde{v}_c$  coincides with  $v_c$  on  $\Omega$ , and equals  $M$ 's expected payoff in a fictional game with a type- $w$  incumbent and challengers drawn according to probability measure  $\omega$  otherwise. The following lemma will be useful for deriving the result.

**Lemma 1**  $\tilde{v}_c(w)$  is non-decreasing in  $w$ .

**Proof.** We show that for any two incumbent types,  $w'$  and  $w''$ , where  $w' < w''$ , there exists a voting strategy  $\tilde{\rho}$  that implements the same policy and re-election outcome for both incumbent types.

For any optimal re-election strategy  $\rho^*(w, g, x)$  we partition  $X$  into two disjoint (possibly empty) subsets;  $\mathcal{P}(w, g) = \{x \in X \mid \rho^*(w, g, x) = 1\}$  and  $\mathcal{C}(w, g) = \{x \in X \mid \rho^*(w, g, x) = 0\}$ . It is then clear that  $G$ 's optimal contract must be of one of two types: (i) offer  $\epsilon$  for choosing  $\tilde{p} = \arg \max_{x \in \mathcal{P}(w, g)} u^G(x)$  and 0 otherwise; or (ii) offer  $w + \delta l_w + \epsilon$  for choosing  $\tilde{c} = \arg \max_{x \in \mathcal{C}(w, g)} u^G(x)$  and 0 otherwise. These contracts result in policy choices of  $\tilde{p}$  and  $\tilde{c}$ , respectively.  $G$  offers the type (i) contract if:

$$u^G(\tilde{p}) > u^G(\tilde{c}) - w - \delta l_w. \quad (11)$$

Because  $P$  can assure herself of re-election whenever  $\mathcal{P}(w, g) \neq \emptyset$ , letting  $\epsilon \rightarrow 0$  it is clear that  $l_w = \frac{\xi w}{1 - \xi \delta}$ , where  $\xi = \gamma(\{g \in \Gamma \mid \mathcal{P}(w, g) \neq \emptyset\})$ .

Let  $x_{w, g}^*$  denote the equilibrium policy with a type- $w$  incumbent and type- $g$  group. There are two cases. First, if  $w'$  and  $g$  are such that  $x_{w', g}^* \in \mathcal{C}(w', g)$ , then we claim  $x_{w', g}^* = g$ . To show this, note that  $g \in \mathcal{C}(w', g)$  implies (by the definition of  $\tilde{c}$ )  $x_{w', g}^* = g$  automatically. Now suppose  $g \in \mathcal{P}(w', g)$ . Then by (11),  $G$  should offer a type (i) contract: contradiction.



To implement the desired outcome with a type- $w''$  incumbent, M can choose  $\bar{\rho}(w'', g, x) = 0$  for all  $x$ . G then offers a contract promising  $\epsilon$  for choosing policy  $g$  and 0 otherwise. Thus she chooses  $g$  and is not re-elected.

Second, if  $w'$  and  $g$  are such that  $x_{w',g}^* \in \mathcal{P}(w', g)$ , then to implement the desired outcome with a type- $w''$  incumbent, M can choose:  $\bar{\rho}(w'', g, x) = 1$  if  $x = x_{w',g}^*$ , and  $\bar{\rho}(w'', g, x) = 0$  otherwise. To show that policy  $x_{w',g}^*$  is chosen, note that  $\bar{\rho}(w'', g, x)$  implies that re-election is feasible for all  $\{g \in \Gamma \mid \mathcal{P}(w', g) \neq \emptyset\}$ ; hence  $l(w''; \{\bar{\rho}(w'', x, g)\}) > l(w'; \{\rho^*(\cdot)\})$ . Further,  $x_{w',g}^* \in \mathcal{P}(w', g)$  implies that (11) holds. This implies that for all  $x \neq x_{w',g}^*$ ,  $u^G(x_{w',g}^*) > u^G(x) - w - \delta l(w''; \{\bar{\rho}(w'', g, x)\})$ . Hence, G offers a type (i) contract  $\beta^*(w', g)$  identical to that offered to a type- $w'$  incumbent. P then receives  $\epsilon + w + \delta l(w''; \{\bar{\rho}(w'', g, x)\})$  for choosing policy  $x_{w',g}^*$  and 0 otherwise. Thus she chooses  $x_{w',g}^*$  and is re-elected. ■

Now consider any optimal stationary contracting equilibrium. Define the set of “average” types as:  $\hat{w} = \inf \{w \mid \bar{v}_c(w) \geq \int_{\Omega} v_c(\bar{w}) \omega(d\bar{w})\}$ . Note that Lemma 1 implies  $\hat{w} \in \text{clos}(\Omega)$ . We use this notation to rewrite (5), which characterizes the re-election choice, as follows. Denote by  $\tilde{x}(w, g)$  the optimal policy that M can induce from P given  $v_c(w)$ , following the approach in (4) (i.e., the optimal policy in  $\mathcal{G}(-w - \delta l_w)$ ). Then M re-elects P of type  $w$  if:

$$u^M(\tilde{x}(w, g)) + \delta v_c(w) > u^M(g) + \delta \bar{v}_c(\hat{w}). \quad (12)$$

By Lemma 1 and the fact that  $u^M(\tilde{x}(w, g)) \geq u^M(g)$  holds trivially, (12) holds for all  $g$  if  $w > \hat{w}$ . The next lemma will be useful for deriving properties of (12).

**Lemma 2**  $u^M(\tilde{x}(w, g))$  is non-decreasing in  $w$ .

**Proof.** Suppose otherwise; i.e.,  $u^M(\tilde{x}(w', g)) > u^M(\tilde{x}(w'', g))$  for some  $g$  and types  $w'$  and  $w''$ , where  $w' < w''$ . By (4), this is possible only if:

$$w' + \delta l_{w'} > w'' + \delta l_{w''}. \quad (13)$$

We begin with two observations: (i) by (4), (13) implies that  $u^M(\tilde{x}(w', g)) \geq u^M(\tilde{x}(w'', g))$  for all  $g$ ; (ii) because  $w' < w''$ , (13) implies that  $\gamma(\Gamma_{w'}) > \gamma(\Gamma_{w''})$ , where  $\Gamma_w = \{g' \in$

$\Gamma \mid (12)$  holds for type  $w$  is the set of group types such that a type- $w$  incumbent is re-elected in equilibrium.

Partition  $\Gamma$  into  $\Gamma^1 = \Gamma_{w'} \cap (\Gamma \setminus \Gamma_{w''})$  and  $\Gamma^2 = \Gamma \setminus \Gamma^1$ . Note that by observation (ii),  $\gamma(\Gamma^1) > 0$ . We now consider the behavior of  $\tilde{v}_c(w)$  over  $\Gamma^1$  and  $\Gamma^2$ . Let  $\hat{v}_c(w, g')$  represent M's discounted expected payoff conditional on a type- $w$  incumbent and type- $g'$  group.

First, for  $g' \in \Gamma^1$ , the definition of  $\Gamma^1$  and (12) imply:

$$u^M(\tilde{x}(w', g')) + \delta v_c(w') \geq u^M(g') + \delta \tilde{v}_c(\hat{w}) > u^M(\tilde{x}(w'', g')) + \delta v_c(w''). \quad (14)$$

Noting that for any  $w$ ,  $\hat{v}_c(w, g') = \max\{u^M(\tilde{x}(w, g')) + \delta \tilde{v}_c(w), u^M(g') + \delta \tilde{v}_c(\hat{w})\}$ , it is clear that  $v_c(w')|_{g' \in \Gamma^1} = \int_{\Gamma^1} \hat{v}_c(w', \tilde{g}) \gamma(d\tilde{g}) > v_c(w'')|_{g' \in \Gamma^1} = \int_{\Gamma^1} \hat{v}_c(w'', \tilde{g}) \gamma(d\tilde{g})$ .

Second, for  $g' \in \Gamma^2$ , we show that with a type- $w'$  incumbent M can guarantee an outcome identical to that with a type- $w''$  incumbent. For any  $g' \in \Gamma^2 \cap (\Gamma \setminus \Gamma_{w''})$  (i.e., where a type- $w''$  incumbent is not re-elected), M may offer  $\rho(w', g', x) = 0$  for all  $x$ . This contract clearly results in no re-election and  $x = g$ , which is also the outcome for a type- $w''$  incumbent. And for any  $g' \in \Gamma^2 \cap \Gamma_{w''}$ , M may offer  $\rho(w', g', x) = 1$  for  $x = \tilde{x}(w'', g')$  and  $\rho(w', g', x) = 0$  otherwise. To show that  $\tilde{x}(w'', g')$  is the outcome, note that  $\tilde{x}(w'', g') = \arg \max_x |_{\rho^*(w'', g', x)=1} u^G(x)$ . Thus,  $u^G(\tilde{x}(w'', g')) > u^G(g') - w'' - \delta l_{w''}$ , which by (13) implies  $u^G(\tilde{x}(w'', g')) > u^G(g') - w' - \delta l_{w'}$  as well. Because M can achieve this outcome in any period, by stationarity,  $v_c(w')|_{g' \in \Gamma^2} \geq v_c(w'')|_{g' \in \Gamma^2}$ .

Finally, note that for any  $w$ ,  $v_c(w) = \gamma(\Gamma^1)v_c(w)|_{g' \in \Gamma^1} + \gamma(\Gamma^2)v_c(w)|_{g' \in \Gamma^2}$ . Combining results, we have  $v_c(w') > v_c(w'')$ , contradicting Lemma 1. ■

To characterize  $w^2$ , note that by (12), for any  $w \leq \hat{w}$ , P is not re-elected if:  $u^M(\tilde{x}(w, g)) - u^M(g) \leq \delta(\tilde{v}_c(\hat{w}) - v_c(w))$ . This condition is satisfied trivially if  $w = \hat{w}$  and  $g = m$ . For  $w < \hat{w}$ , the right-hand side of this condition is positive and by Lemma 1 non-increasing in  $w$ . Additionally, by Lemma 2, the left-hand side is non-negative and non-decreasing in  $w$ . Thus for all  $w \leq \hat{w}$ , there exists a non-empty set of group types  $\mathcal{D}_w$  satisfying the condition, where  $\mathcal{D}_{w''} \subseteq \mathcal{D}_{w'}$  for  $w'' > w'$ . Thus,  $w^2 = \sup\{w \in \Omega \mid \exists g \in \Gamma \cap \mathcal{D}_w\}$ , and 0 otherwise.

To characterize  $w^1$ , (12) implies P is never re-elected if for all  $g$ :  $u^M(\tilde{x}(w, g)) - u^M(g) \leq$

$\delta(\bar{v}_c(\hat{w}) - v_c(w))$ . Because this condition is stricter than that for  $w^2$ , it is clear that  $w^1 \leq w^2$ . To show existence of a  $w$  satisfying the condition, consider  $w = 0$ . Clearly,  $l_0 = 0$  for any  $\rho$ ; hence,  $\bar{x}(w, g) = g$  and the left-hand side of the condition is zero. Additionally,  $\bar{v}_c(\hat{w}) \geq \bar{v}_c(0)$ , and therefore the right-hand side is positive and the condition holds at  $w = 0$ . Because  $\mathcal{D}_{w''} \subseteq \mathcal{D}_{w'}$  for  $w'' > w'$ ,  $w^1 = \sup\{w \in \Omega \mid \Gamma \subseteq \mathcal{D}_w\}$ , and 0 otherwise. Putting all of the derived inequalities together,  $\hat{w} \geq w^2 \geq w^1 \geq 0$ .

To characterize the re-election function  $\eta$  for  $w \in [w^1, w^2]$ , note that by the definition of  $\mathcal{D}_w$  realized re-election choices are given by:  $\eta(w, g) = 1(g \notin \mathcal{D}_w)$ . Two comparative statics follow. First, note that the left-hand side of (12) is non-decreasing in  $w$ . Thus, given  $g$  and  $w'' > w'$ , if (12) holds for a type- $w'$  incumbent, then it also holds for a type- $w''$  incumbent. Second, if  $u^M$  is concave, then  $u^M(\bar{x}(w, g)) - u^M(g)$  is increasing in  $|m - g|$ . Thus, for any  $g', g''$  such that  $|m - g''| > |m - g'|$ , if (12) holds for a type- $g'$  group, then it also holds for a type- $g''$  group; that is,  $g' \notin \mathcal{D}_w$  implies  $g'' \notin \mathcal{D}_w$ . Therefore,  $\eta(w, g)$  is non-decreasing in  $w$ , and non-decreasing in  $g$  if  $u^M$  is concave. ■

**Proof of Comment 1.** Suppose otherwise. Let  $\bar{v}_s \equiv \int_{\Omega} v_s(\bar{w})\omega(d\bar{w})$  represent the discounted expected payoff from electing a challenger. Then for some  $w' \in \Omega$ , either (i)  $v_s(w') > \bar{v}_s$ , or (ii)  $v_s(w') < \bar{v}_s$ . Suppose that (i) holds. By (5),  $\rho_t^*(w', g_t, b_t, x_t) = 1$  for all  $g_t, b_t$ , and  $x_t$ . Given this voting strategy,  $G_t$ 's best response is:  $\beta_t^*(w', g_t) = \epsilon$  if  $x_t = g_t$ , and  $\beta_t^*(w', g_t) = 0$  otherwise, for some  $\epsilon > 0$  (i.e., a contract of the form in (1)). Clearly, this induces policy choice  $\chi_t^*(w', g_t, b_t) = g_t$ . Note that retaining a type- $w'$  politician in each period implies  $v_s(w') = \frac{1}{1-\delta} \int_{\Gamma} u^M(\bar{g})\gamma(d\bar{g})$ . But in any period  $t$ ,  $M$  can achieve at least  $u^M(g_t)$  by adopting strategy  $\rho_t^*(\cdot) = 1$  or all  $g_t, b_t$ , and  $x_t$ ; thus,  $\bar{v}_s \geq \frac{1}{1-\delta} \int_{\Gamma} u^M(\bar{g})\gamma(d\bar{g})$ . Hence,  $\bar{v}_s \geq v_s(w')$ : contradiction.

Now suppose that (ii) holds. Let  $\underline{\mathcal{W}} \equiv \{\bar{w} \mid v_s(\bar{w}) < \bar{v}_s\}$  denote the set of “below average” types. If  $\omega(\underline{\mathcal{W}}) > 0$ , then there exists a non-empty set of types  $\{\bar{w}' \mid v_s(\bar{w}') > \bar{v}_s\}$ . By part (i), this set of types is empty: contradiction. We conclude that  $\omega(\underline{\mathcal{W}}) = 0$ . ■

**Proof of Proposition 2.** We first prove that any strategy profile in  $\mathcal{E}_s$  is an equilibrium,

and then prove optimality. For M, since P's policy choice is constant with respect to the incumbent type  $w$ , she is indifferent amongst all candidates. Thus, any voting strategy  $\rho_t$  is optimal. We fix  $\rho_t^*$  as follows:

$$\rho_t^*(w, g_t, b_t, \chi_t^*) = \begin{cases} 1 & \text{if } m \notin \mathcal{G}_t(-\frac{w}{1-\delta}) \text{ and } u^M(x_t) \geq u^M(\hat{x}_t), \\ & \text{or } m \in \mathcal{G}_t(-\frac{w}{1-\delta}) \text{ and } x_t = m \\ 0 & \text{otherwise,} \end{cases}$$

where  $\hat{x}_t = \arg \max_{x_t \in \mathcal{G}_t(-\frac{w}{1-\delta})} u^M(x_t)$ . Generally,  $\rho_t^*(\cdot)$  requires that re-election occur only if  $G_t$  chooses the optimal policy for M within  $\mathcal{G}_t(-\frac{w}{1-\delta})$ . If  $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$ ,  $G_t$  must choose  $m$ ; otherwise, she must choose  $\hat{x}_t$ , which by the single-peakedness of  $u^M$  is either  $\underline{g}_t(-\frac{w}{1-\delta})$  or  $\bar{g}_t(-\frac{w}{1-\delta})$  (though we allow re-election in the out of equilibrium event that P chooses a better policy).

Now consider  $G_t$ 's incentives. Given  $\rho_t^*$  and  $\chi_t^*$ , if  $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$ , then  $G_t$  must offer at least  $\frac{w}{1-\delta}$  to induce P to choose any policy other than  $m$ . But since  $u^{G_t}(g_t) - u^{G_t}(m) < \frac{w}{1-\delta}$  by construction of  $\mathcal{G}_t(\cdot)$ , she is unwilling to do so and thus optimally offers  $\beta^*(w, g_t) = b_t(x_t) = 0$ . If  $m \notin \mathcal{G}_t(-\frac{w}{1-\delta})$ , then for all  $x_t \in \mathcal{G}_t(-\frac{w}{1-\delta}) \setminus \hat{x}_t$ , P is not re-elected and  $u^{G_t}(g_t) - u^{G_t}(m) \leq \frac{w}{1-\delta}$ , so  $G_t$  does not offer  $\frac{w}{1-\delta}$  for such  $x_t$ . Clearly,  $\hat{x}_t = \arg \max_{x_t \in \bar{\mathcal{G}}_t(\cdot) \cup \hat{x}_t} u^{G_t}(x_t)$ , and P chooses  $\hat{x}_t$  if  $b_t(\hat{x}_t) > b_t(x_t)$  for all  $x_t \neq \hat{x}_t$ . Thus  $G_t$  optimally offers:

$$\beta^*(w, g_t) = b_t(x_t) = \begin{cases} \epsilon & \text{if } x_t = \hat{x}_t \\ 0 & \text{otherwise,} \end{cases}$$

for some  $\epsilon > 0$ . Letting  $\epsilon \rightarrow 0$ , we obtain the optimal contract.

Next, given  $\rho_t^*$  and  $\beta^*$ , if  $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$  P receives  $\frac{w}{1-\delta} + \epsilon$  for choosing  $m$  and zero otherwise. If  $m \notin \mathcal{G}_t(-\frac{w}{1-\delta})$ , then P receives  $\frac{w}{1-\delta} + \epsilon$  for choosing  $\hat{x}_t$  and either zero or  $\frac{w}{1-\delta}$  otherwise. Thus her strategy is  $\chi_t^*$  as specified.

To prove optimality, suppose that there exist stationary equilibria  $\mathcal{E}'_s$  with strictly higher payoffs for M. Note that by Comment 1,  $v_s(w)$  is constant with respect to  $w$ , and that in  $\mathcal{E}_s$  M receives her ideal policy whenever  $m \in \mathcal{G}_t(-\frac{w}{1-\delta})$ . Hence in  $\mathcal{E}'_s$  for a type- $w$  incumbent there exists some  $g_t$  such that  $m \notin \mathcal{G}_t(-\frac{w}{1-\delta})$ ,  $\chi_t^*(w, g_t, b_t) > \bar{g}_t(-\frac{w}{1-\delta})$  ( $< \underline{g}_t(-\frac{w}{1-\delta})$ ) for  $m > (<) g_t$ . Assume without loss of generality that  $g_t > m$ , so that  $\chi_t^*(w, g_t, b_t) > \bar{g}_t(-\frac{w}{1-\delta})$ . Since  $\chi_t^*(w, g_t, b_t) \notin \mathcal{G}_t(-\frac{w}{1-\delta})$ ,  $u^{G_t}(\chi_t^*(w, g_t, b_t)) < -\frac{w}{1-\delta}$ . Let  $G_t$  replace its equilibrium

contract with:  $b_t(x_t) = -\frac{w}{1-\delta} + \epsilon$  if  $x_t = g_t$ , and zero otherwise. Then, letting  $\epsilon \rightarrow 0$ , P's dominant policy choice is  $g_t$ , and  $G_t$  receives  $-\frac{w}{1-\delta}$ . Thus,  $\chi_t^*(w, g_t, b_t)$  cannot be an equilibrium policy: contradiction. ■

**Proof of Proposition 3.** We proceed in two steps. First, we establish the optimality of  $\mathcal{E}_c$  as a cooperative phase. Suppose otherwise; i.e., that M attains higher discounted expected utility under another set of cooperative phase strategy profiles  $\hat{\mathcal{E}}$ . Let  $\{\hat{\rho}_t(\cdot)\}$  denote M's voting strategy under  $\hat{\mathcal{E}}$ , and let  $\{\hat{\beta}_t(\cdot)\}$  and  $\{\hat{\chi}_t(\cdot)\}$  denote  $G_t$  and P's strategies, respectively. It is clear that to be a cooperative phase,  $\{\hat{\beta}_t(\cdot)\}$  and  $\{\hat{\chi}_t(\cdot)\}$  must be best responses each other as well as to  $\{\hat{\rho}_t(\cdot)\}$ . Suppose that in each period  $t$  of the contracting game M offers the contract  $\rho^c(\cdot) = \hat{\rho}_t(h_t, \cdot)$  for all  $h_t \in H_t$  in the cooperative phase. This contract strategy is stationary because  $\hat{\rho}_t(h'_t, \cdot) = \hat{\rho}_t(h''_t, \cdot)$  for all  $h'_t, h''_t \in H_t$  in the cooperative phase. It is straightforward to verify that since  $\hat{\beta}_t(\cdot)$  and  $\hat{\chi}_t(\cdot)$  are best responses to  $\hat{\rho}_t(\cdot)$  in a cooperative phase, they are also best responses to  $\rho^c(\cdot)$ . Since  $\hat{\beta}_t(\cdot)$  and  $\hat{\chi}_t(\cdot)$  cannot induce a punishment phase (by assumption), they induce the same continuation games under  $\rho^c(\cdot)$  and  $\hat{\rho}_t(\cdot)$ . Therefore, the policies realized under the stationary contracts  $\{\rho^c(\cdot)\}$  are identical to those under  $\hat{\mathcal{E}}$  for all  $h_t \in H_t$ . This contradicts the optimality of  $\mathcal{E}_c$  in the contracting game.

Second, we show that trigger strategies of the form  $(\mathcal{E}_c, \mathcal{E}_0, n)$  constitute a subgame perfect equilibrium for sufficiently large  $n$ . In the punishment phase, no action can affect the duration of the phase. For any period  $t$  in a punishment phase, given  $\chi_t^*(\cdot) = g_t$ , M is indifferent between electing and re-electing, so chooses  $\rho_t^*(\cdot) = 0$ . Given  $\rho_t^*(\cdot) = 0$ , a bribing strategy  $\beta_t^*(\cdot)$  as in (1), and  $\chi_t^*(\cdot) = g_t$  are clearly Nash.

Now consider the cooperative phase. Because  $\beta_t(\cdot)$  and  $\chi_t(\cdot)$  are best responses to  $\{\rho_t(\cdot)\}$  in  $\mathcal{E}_c$ , we need check only that M casts re-election votes in accordance with  $\mathcal{E}_c$ . M will not re-elect an incumbent in equilibrium if:

$$\int_{\Omega} v_c(\tilde{w})\omega(d\tilde{w}) \geq \sum_{j=0}^{n-1} \delta^j Eu^M(g_t) + \delta^n \int_{\Omega} v_c(\tilde{w})\omega(d\tilde{w}). \quad (15)$$

Since  $u^M(\tilde{x}_t(w, g_t)) \geq u^M(g_t)$  trivially, this condition holds. Next, M re-elects an incumbent

in equilibrium if:

$$v_c(w) \geq \sum_{j=0}^{n-1} \delta^j \text{Eu}^M(g_t) + \delta^n \int_{\Omega} v_c(\tilde{w}) \omega(d\tilde{w}). \quad (16)$$

Clearly, under  $\mathcal{E}_c$ ,  $v_c(w) \geq \sum_{j=0}^{\infty} \delta^j \text{Eu}^M(g_t)$  for all  $w \in \Omega$ . Thus, there exists some  $n^*$  (possibly infinite) such that (16) holds for all  $n > n^*$ . For all such  $n$ ,  $\mathcal{E}_c$  is sustainable as the cooperative phase of a trigger equilibrium. ■

**Proof of Proposition 4.** (i) Suppose otherwise. Then there exist  $w', w'' \in \Omega$ , ( $w' < w''$ ) such that  $v_T(w', \theta) > v_T(w'', \theta)$ . Let  $\rho^*(w, \theta, g_t, x)$  represent the optimal voting contract offered to a type- $(w, \theta)$  incumbent for choosing policy  $x$  when  $G_t$  has ideal point  $g_t$ . Let  $x_{w, \theta, g_t}^*$  denote the policy chosen for an arbitrary  $P$  and  $G_t$ . Suppose that for all  $\theta, g_t$ , and  $x$ ,  $M$  offers the type- $w''$  incumbent the following contract:

$$\hat{\rho}(w'', \theta, g_t, x) = \begin{cases} 1 & \text{if } x = x^*(w', \theta, g_t) \text{ and } \rho^*(w', \theta, g_t, x_{w', \theta, g_t}^*) = 1 \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

That is,  $M$  re-elects the type- $(w'', \theta)$  incumbent if and only if she chooses the equilibrium policy of a type- $(w', \theta)$  incumbent, and the type- $(w', \theta)$  incumbent is offered re-election according to her optimal contract.

It is clear that under  $\hat{\rho}(\cdot)$ , the type- $(w'', \theta)$  incumbent is not re-elected whenever a type- $(w', \theta)$  incumbent is not re-elected. In these cases  $G_t$  can write a contract of the form in (1), thus obtaining  $x_t = g_t$ .

Now consider cases in which a type- $(w', \theta)$  incumbent is re-elected. By choosing policy  $x_{w', \theta, g_t}^*$  whenever possible, the type- $(w'', \theta)$  incumbent assures herself of an expected payoff of  $\hat{l}_{w'', \theta} \equiv \sum_{i=\theta}^T \delta^{i-\theta} \gamma(\Gamma_{w', i}) w'' > 0$ , where  $\Gamma_{w', i} \equiv \{g \mid \rho^*(w', \theta, g, x_{w', \theta, g}^*) = 1\}$  is the set of group types such that a type- $(w', \theta)$  incumbent is re-elected. By not choosing  $x_{w', \theta, g_t}^*$ ,  $M$  receives zero.  $G_t$  may induce the type- $(w'', \theta)$  incumbent to deviate from  $x_{w', \theta, g_t}^*$  only by offering some contract  $\hat{b}(x)$  satisfying  $\hat{b}(x) \geq w'' + \delta l_{w'', \theta}$  for some  $x \neq x_{w', \theta, g_t}^*$ . This implies  $u^{G_t}(x) - w'' - \delta \hat{l}_{w'', \theta} > u^{G_t}(x_{w', \theta, g_t}^*)$ . But then  $G_t$  could have offered  $w' + \delta l_{w', \theta}$  to the type- $(w', \theta)$  incumbent to achieve policy  $x$ , where  $l_{w', \theta} \equiv \sum_{i=\theta}^T \delta^{i-\theta} \gamma(\Gamma_{w', i}) w'$  is a type- $(w', \theta)$  incumbent's expected lifetime payoff. Since  $w' + \delta l_{w', \theta} < w'' + \delta \hat{l}_{w'', \theta}$ , this implies that  $x_{w', \theta, g_t}^*$  could not



have been chosen by a type- $(w', \theta)$  incumbent. Thus  $x_{w', \theta, g_t}^*$  is feasibly implementable by a type- $(w'', \theta)$  incumbent.

We conclude that in an optimal contracting equilibrium, a type- $(w'', \theta)$  incumbent can be induced to choose policies at least as good as those chosen by a type- $(w', \theta)$  incumbent. Thus,  $v_T(w', \theta) \leq v_T(w'', \theta)$ : contradiction.

(ii) Suppose otherwise. Then there exist  $\theta', \theta'' \in \{1, \dots, T\}$ , ( $\theta' < \theta''$ ) such that  $v_T(w, \theta') < v_T(w, \theta'')$ . This proof is identical to that in part (i), but with  $\hat{\rho}(\cdot)$  redefined as follows:

$$\hat{\rho}(w, \theta', g_t, x) = \begin{cases} 1 & \text{if } x = x_{w, \theta'', g_t}^* \text{ and } \rho^*(w, \theta'', g_t, x_{w, \theta'', g_t}^*) = 1 \\ 0 & \text{otherwise.} \end{cases} \quad (18)$$

(iii) By part (ii),  $v_T(w, \theta) \leq v_T(w, \infty)$  for any  $w$ . Since  $v_T(w, \infty) = v_c(w)$ ,  $v_T(w, \theta) \leq v_c(w)$  for all  $w \in \Omega$ . ■

**Proof of Proposition 5.** Let  $\Gamma_{w, \theta} \equiv \{g \mid \rho^*(w, \theta, g, x_{w, \theta, g}^*) = 1\}$  denote the set of group types such that a type- $(w, \theta)$  incumbent is re-elected. The *ex ante* re-election probability for a type- $(w, \theta)$  incumbent is then  $\gamma(\Gamma_{w, \theta})$ . We claim that in an optimal equilibrium for M,  $\Gamma_{w, \theta} \subseteq \Gamma_{w, \theta-1}$  for all  $\theta$  ( $1 < \theta \leq T$ ).

We proceed via induction on the set  $\{\Gamma_{w, \theta} \mid T - \tau \leq \theta \leq T\}$ . Clearly,  $\Gamma_{w, \theta} \subseteq \Gamma_{w, \theta-1}$  for all  $\theta$  when  $\tau = 1$ . Suppose that  $\Gamma_{w, \theta} \subseteq \Gamma_{w, \theta-1}$  for all  $\theta$  and some  $\tau$  ( $1 < \tau < T$ ). If  $\Gamma_{w, \theta-1} = \emptyset$ , then  $\Gamma_{w, \theta-1} \subseteq \Gamma_{w, \theta-2}$  trivially and the result is established. Otherwise, by (9), for any  $g \in \Gamma_{w, \theta-1}$ , the induction hypothesis implies:

$$u^M(\tilde{x}(w, \theta-1, g)) + \delta v_T(w, \theta) \geq u^M(g) + \delta \int_{\Omega} v_T(\tilde{w}, 1) \omega(d\tilde{w}). \quad (19)$$

To show that a type- $(w, \theta-2)$  incumbent is re-elected when  $G_t$  is of type  $g$ , it is sufficient to establish that  $u^M(\tilde{x}(w, \theta-2, g)) + \delta v_T(w, \theta-1) \geq u^M(\tilde{x}(w, \theta-1, g)) + \delta v_T(w, \theta)$ . By Proposition 4(ii),  $v_T(w, \theta-1) \geq v_T(w, \theta)$ . Thus it is sufficient to show that  $u^M(\tilde{x}(w, \theta-2, g)) \geq u^M(\tilde{x}(w, \theta-1, g))$ . By (8), this is true if  $w + \delta l_{w, \theta-1} \geq w + \delta l_{w, \theta}$ , or equivalently:  $l_{w, \theta-1} \geq l_{w, \theta}$ . Expanding terms, this is:

$$\sum_{k=0}^{T-\theta+1} \delta^k w \gamma(\Gamma_{w, k+\theta-1}) \geq \sum_{k=0}^{T-\theta} \delta^k w \gamma(\Gamma_{w, k+\theta}), \quad (20)$$

which holds if  $\gamma(\Gamma_{w,k+\theta-1}) > \gamma(\Gamma_{w,k+\theta})$  over  $0 \leq k \leq T-\theta$ . This is implied by the induction hypothesis. Therefore,  $\Gamma_{w,\theta-1} \subseteq \Gamma_{w,\theta-2}$  and the induction hypothesis holds for  $\tau+1$ . ■

**Proof of Proposition 6.** Optimality follows from an analogous argument to that in the proof of Proposition 3.

We show that trigger strategies of the form  $(\mathcal{E}_T, \mathcal{E}_0, n)$  constitute a subgame perfect equilibrium for sufficiently large  $n$ . In the punishment phase, no action can affect the duration of the phase. For any period  $t$  in a punishment phase, given  $\chi_t^*(\cdot) = g_t$ , M is indifferent between electing and re-electing, so M chooses  $\rho_t^*(\cdot) = 0$ . Given  $\rho_t^*(\cdot) = 0$ , a bribing strategy  $\beta_t^*(\cdot)$  as in (1), and  $\chi_t^*(\cdot) = g_t$  are clearly Nash.

Now consider the cooperative phase. Because  $\beta_t(\cdot)$  and  $\chi_t(\cdot)$  are best responses to  $\{\rho_t(\cdot)\}$  in  $\mathcal{E}_T$ , we need check only that M casts re-election votes in accordance with  $\mathcal{E}_T$ . M will not re-elect an incumbent in equilibrium if:

$$\int_{\Omega} v_T(\tilde{w}, 1) \omega(d\tilde{w}) \geq \sum_{j=0}^{n-1} \delta^j \text{Eu}^M(g_t) + \delta^n \int_{\Omega} v_T(\tilde{w}, 1) \omega(d\tilde{w}). \quad (21)$$

Since  $u^M(\tilde{x}_t(w, g_t)) \geq u^M(g_t)$  trivially, this condition holds. Next, M re-elects a type- $(w, \theta)$  incumbent in equilibrium if:

$$v_T(w, \theta+1) \geq \sum_{j=0}^{n-1} \delta^j \text{Eu}^M(g_t) + \delta^n \int_{\Omega} v_T(\tilde{w}, 1) \omega(d\tilde{w}). \quad (22)$$

Clearly, under  $\mathcal{E}_T$ ,  $v_T(w, \theta+1) \geq \sum_{j=0}^{\infty} \delta^j \text{Eu}^M(g_t)$  for all  $w \in \Omega$  and  $\theta$ . Thus, there exists some  $n^*$  (possibly infinite) such that (22) holds for all  $n > n^*$ . For all such  $n$ , strategy profiles in  $\mathcal{E}_T$  are sustainable as the cooperative phase of a trigger equilibrium. ■

## REFERENCES

- Austen-Smith, David. 1987. "Interest Groups, Campaign Contributions, and Probabilistic Voting." *Public Choice* 54: 123-139.
- Austen-Smith, David, and Jeffrey S. Banks. 1989. "Electoral Accountability and Incumbency." in Peter Ordeshook (ed.), *Models of Strategic Choice in Politics*. Ann Arbor: MI, University of Michigan Press.
- Banks, Jeffrey S., and Rangarajan Sundaram. 1993. "Adverse Selection and Moral Hazard in a Repeated Elections Model." in W. Barnett et al. (eds.), *Political Economy: Institutions, Information, Competition and Representation*. New York: NY, Cambridge University Press.
- Banks, Jeffrey S., and Rangarajan Sundaram. 1998. "Optimal Retention in Agency Problems." *Journal of Economic Theory* 82: 293-323.
- Barganza, Juan Carolos. 2000. "Two Roles for Elections: Disciplining the Incumbent and Selecting a Competent Candidate." *Public Choice* 105: 165-193.
- Baron, David P. "Electoral Competition with Informed and Uninformed Voters." *American Political Science Review* 88: 33-47.
- Barro, Robert. 1973. "The Control of Politicians: An Economic Model." *Public Choice* 14: 19-42.
- Besley, Timothy. 2003. "Principled Agents? Motivation and Incentives in Government." Unpublished manuscript.
- Besley, Timothy, and Michael Smart. 2001. "Does Tax Competition Raise Voter Welfare?" Unpublished manuscript.
- Calvert, Randall L. 1986. *Models of Imperfect Information in Politics*. Chur, Switzerland: Harwood Academic Publishers.
- Denzau, Arthur T., and Michael C. Munger. 1986. "Legislators and Interest Groups: How Unorganized Groups Get Represented." *American Political Science Review* 80: 89-106.
- Dutta, Prajit K. 1995. "A Folk Theorem for Stochastic Games." *Journal of Economic Theory* 66: 1-32.
- Faure-Grimaud, Antoine, and David Martimort. 2003. "Regulatory Inertia." *RAND Journal of Economics* 34(3): 413-437.
- Fearon, James D. 1999. "Electoral Accountability and the Control of Politicians: Selecting Good Types Versus Sanctioning Poor Performance." In Bernard Manin, Adam Przeworski, Susan Stokes, (eds.), *Democracy, Accountability and Representation*. Cambridge: Cambridge University Press.

- Ferejohn, John. 1986. "Incumbent Performance and Electoral Control." *Public Choice* 50: 5-25.
- Grossman, Gene, and Elhanan Helpman. 1994. "Protection for Sale." *American Economic Review* 84: 833-850.
- Grossman, Gene, and Elhanan Helpman. 1996. "Electoral Competition and Special Interest Politics." *Review of Economic Studies* 63: 265-286.
- Grossman, Gene, and Elhanan Helpman. 2001. *Special Interest Politics*. Cambridge, MA: MIT Press.
- Harrington, Joseph. 1993. "Economic Policy, Economic Performance, and Elections." *American Economic Review* 83: 27-42.
- Hindriks, Jean, and Paul Belleflamme. 2001. "Yardstick Competition and Political Agency Problems." Queen Mary and Westfield College, Department of Economics Discussion Papers, No. 444.
- Le Borgne, Eric, and Ben Lockwood. 2001a. "Do Elections Always Motivate Incumbents?" Warwick Economic Research Paper No. 580.
- Le Borgne, Eric, and Ben Lockwood. 2001b. "Candidate Entry, Screening, and the Political Budget Cycle." Unpublished manuscript.
- Lopez, Edward J. 2002. "Congressional Voting on Term Limits." *Public Choice* 112(3-4): 405-431.
- Martimort, David. 1999. "The Life Cycle of Regulatory Agencies." *Review of Economic Studies* 66(4): 929-947.
- Peltzman, Sam. 1976. "Toward a More General Theory of Economic Regulation." *Journal of Law and Economics* 19: 211-240.
- Persson, Torsten, and Guido Tabellini. 2000. *Political Economics: Explaining Economic Policy*. Cambridge, MA: MIT Press.
- Rogoff, Kenneth. 1990. "Equilibrium Political Budget Cycles." *American Economic Review* 80: 21-36.
- Rogoff, Kenneth, and Anne Sibert. 1988. "Elections and Macroeconomic Policy Cycles." *Review of Economic Studies* 55: 1-16.
- Salant, David J. 1995. "Behind the Revolving Door: A New View of Public Utility Regulation." *RAND Journal of Economics* 26(3): 362-377.
- Schlozman, Kay Lehman, and John T. Tierney. 1986. *Organized Interests and American Democracy*. New York: Harper and Row.

- Smart, Michael, and Daniel Sturm. 2003. "Does Democracy Work? Estimating Incentive and Selection Effects of U.S. Gubernatorial Elections, 1950-2000." Unpublished manuscript.
- Smart, Michael, and Daniel Sturm. 2004. "Term Limits and Electoral Accountability." Unpublished manuscript.
- Stigler, George. 1971. "The Theory of Economic Regulation." *Bell Journal of Economics* 2: 3-21.







APR 27 2005

Date Due

Lib-26-67

MIT LIBRARIES



3 9080 02618 3209

